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LOW LEVEL WIND MEASUREMENTS FOR BALLISTIC MISSILE APPLICATION

PREPARED BY

MISSILE METEOROLOGY DIVISION

U. S. ARMY
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WHITE SANDS MISSILE RANGE
NEW MEXICO

MISSILE METEOROLOGY DIVISION

LOW LEVEL WIND MEASUREMENTS

FOR BALLISTIC MISSILE APPLICATION

BY: ROBERT N. SWANSON

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APPROVED:

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A B S T R A C T

APPLICATION OF THE LOW LEVEL WIND TURBULENCE SPECTRUM TO THE BALLISTIC MISSILE PROBLEM WITH THE AID OF TAYLOR'S HYPOTHESIS IS DISCUSSED. RESULTS OF CROSS-SPECTRAL ANALYSIS ARE DISCUSSED WITH REFERENCE TO PREDICTING THE WIND AT THE LAUNCHER FROM A SENSOR AT SOME POINT IN SPACE AWAY FROM THE LAUNCHER. EXAMPLES OF SPECTRAL AND CROSS-SPECTRAL ESTIMATES ARE PRESENTED.

CONTENTS

INTRODUCTION

The problem of obtaining the best possible measurement of the low level wind for ballistic missile application has not been resolved to the satisfaction of either the meteorologist or the ballistician. This wind problem is separated into two areas, the preshoot launcher setting for wind effect and the post launch analysis of the missile trajectory and impact.

The low level wind measurements are taken with tower mounted sensors and/or balloons. These sensors are located from 50 to 2,000 feet from the missile launcher.

Present methods of estimating the ballistic wind utilize short period (i.e., instantaneous to about one-minute averages) wind data prior to and at launch time. More accurate pre-launch estimates of launch time wind and missile dispersion patterns can be made by considering more stable estimates of mean wind (i.e., 10- to 60-minute averages), and also analyzing the wind data to obtain spectral and cross-spectral characteristics of the turbulent wind field.

DISCUSSION

Eulerian time correlation considers velocity fluctuations at a fixed point with respect to time. Eulerian space correlation considers simultaneous values of velocity fluctuations at two or more points in space with respect to the distance between the points.

Nearly all wind measurements are of the Eulerian-time form, but wind fluctuations, as they affect missiles in flight, are of the Eulerian-space form, i.e., turbulence described in forms of wave length or wave number in an effectively frozen atmosphere. To resolve this we use G. I. Taylor's hypothesis (Ref. 1), which transforms a time series into a space series. The hypothesis is: "If u, the component at a fixed point of turbulence motion in the direction of the main stream in a wind tunnel, is resolved into harmonic components, the mean value of u^2 may be regarded as being the sum of contributions from all frequencies. If $u^2F(n)\,dn$ is the contribution from frequencies between n and $n + dn$, then

$$\int_0^\infty F(n)\,dn = 1 \qquad (1)$$

"If $F(n)$ is plotted against n, the diagram so produced is a form of the spectrum curve.-------

"It is clear that when the eddies are large the correlation Rx between simultaneous values of u at distance x apart must fall away with increasing x more slowly than when eddies are small. One may therefore anticipate that when the (Rx, x) curve has a small spread in the x co-ordinate the F(n) curve will extend to large values of n or vice versa.

"If the velocity of the air stream which carries the eddies is very much greater than the turbulent velocity, one may assume that the sequence of changes in u at the fixed point are simply due to the passage of an unchanging pattern of turbulent motion over the point, i.e., one may assume that

$$u = \phi(t) = \phi\left(\frac{-x}{U}\right), \qquad (7)$$

where x is measured upstream at time t = 0 from the fixed point where u is measured. In the limit when $u/U \longrightarrow 0$ (7) is certainly true. Assuming that (7) is still true when u/U is small but not zero, Rx is defined as

$$Rx = \frac{\overline{\phi(t)\,\phi\left(t + \frac{x}{U}\right)}}{\overline{u^2}} \qquad (8)"$$

U = mean wind stream velocity.

From this is derived the relationship

$$Rx = \frac{\overline{\phi(t)\,\phi\left(t + \frac{x}{U}\right)}}{\overline{u^2}} = \int_0^\infty F(n) \cos \frac{2\pi n x}{U} \, dn$$

and using the Fourier integral theorem the following equation is found for F(n)

$$F(n) = \frac{4}{U} \int_0^\infty Rx \cos \frac{2\pi n x}{U} \, dx .$$

Thus if F(n) is known Rx may be calculated or vice versa.

Taylor's hypothesis has been proven experimentally in wind tunnels.

OTHER EXPERIMENTS HAVE DEMONSTRATED THAT THE HYPOTHESIS $x = Ut$ HOLDS TRUE IN THE ATMOSPHERE FOR WAVELENGTHS OF LESS THAN 1,000 FEET (REF. 2, 3, 4, 5). SUFFICIENT EXPERIMENTS HAVE NOT BEEN MADE TO VERIFY THE HYPOTHESIS FOR WAVELENGTHS IN EXCESS OF 1,000 FEET IN THE ATMOSPHERE.

POWER SPECTRAL ESTIMATES OF TURBULENCE STRUCTURE

IN RECENT YEARS CONSIDERABLE WORK HAS BEEN DONE ON THE EVALUATION OF THE POWER SPECTRAL ESTIMATES OF THE TURBULENT WIND FIELD AT A SINGLE POINT. SINCE THIS CAN BE TRANSFORMED INTO SPACE SPECTRAL ESTIMATES BY TAYLOR'S HYPOTHESIS, THE SPECTRAL ESTIMATES OF TIME SERIES SHALL BE DISCUSSED FIRST. IN ALL FOLLOWING EXAMPLES THE CRITERION OF u/U BEING SMALL IS APPROXIMATELY MET, I.E., GUSTINESS RATIO $(\sigma_u/U) < 0.3$.

HUSS AND BUSHNELL (REF. 6) CALCULATED THE POWER SPECTRUM "---- BY FINDING THE VARIANCE OF VELOCITY $\sigma^2_{(i/j)}$ AS A FUNCTION OF i AND j, TWO LENGTHS OF TIME INTERVALS, AND THEN DIVIDING BY THE CORRESPONDING BANDWIDTH

$\left[\frac{1}{2i} - \frac{1}{2j}\right]$ AND FINALLY ASSIGNING THE RESULT TO THE MIDFREQUENCY

$$\frac{1}{2}\left[\frac{1}{2i} + \frac{1}{2j}\right]$$ ---------."

THE VARIANCES WERE FOUND FROM THE RELATION

$$\sigma^2_{(i/j)} = \sigma^2_{(i/L)} - \sigma^2_{(j/L)} .$$

L = FUNDAMENTAL SAMPLE PERIOD.

SPECTRAL ESTIMATES MAY BE NORMALIZED BY DIVIDING BY THE SQUARE OF THE MEAN WIND SPEED.

FIGURES 1 AND 2 ARE SPECTRAL ESTIMATES OF THE WIND 15 FEET AND 180 FEET ABOVE THE SURFACE AT WHITE SANDS MISSILE RANGE COMPUTED USING THE METHOD DESCRIBED BY HUSS AND BUSHNELL.

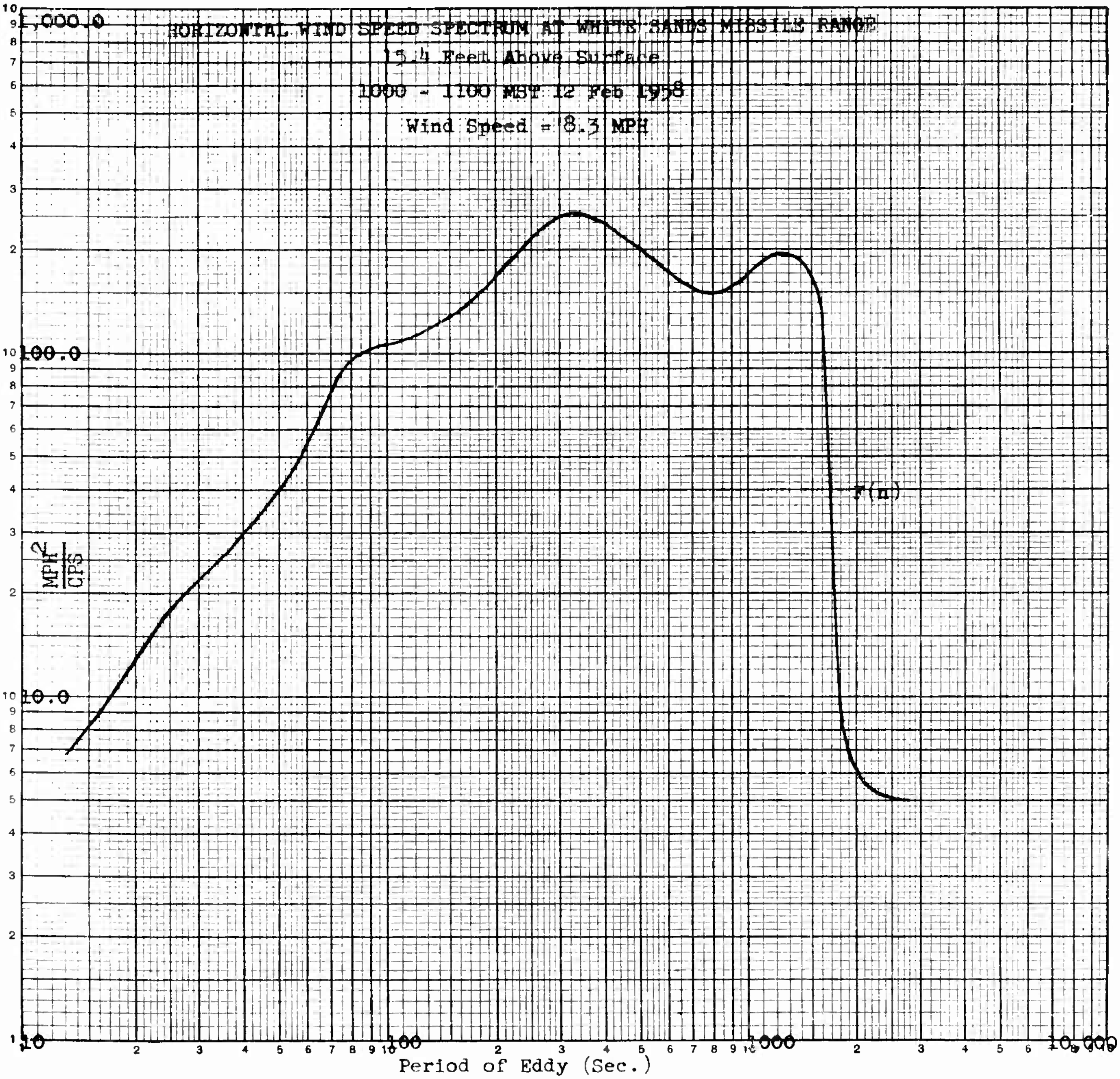

HORIZONTAL WIND SPEED SPECTRUM AT WHITE SANDS MISSILE RANGE
15.4 Feet Above Surface
1000 - 1100 MST 12 Feb 1958
Wind Speed = 8.3 MPH
1,000.0
100.0
10.0
10
100
1000
10,000
$\frac{MPH^2}{CPS}$
F(n)
Period of Eddy (Sec.)

Fig. 1

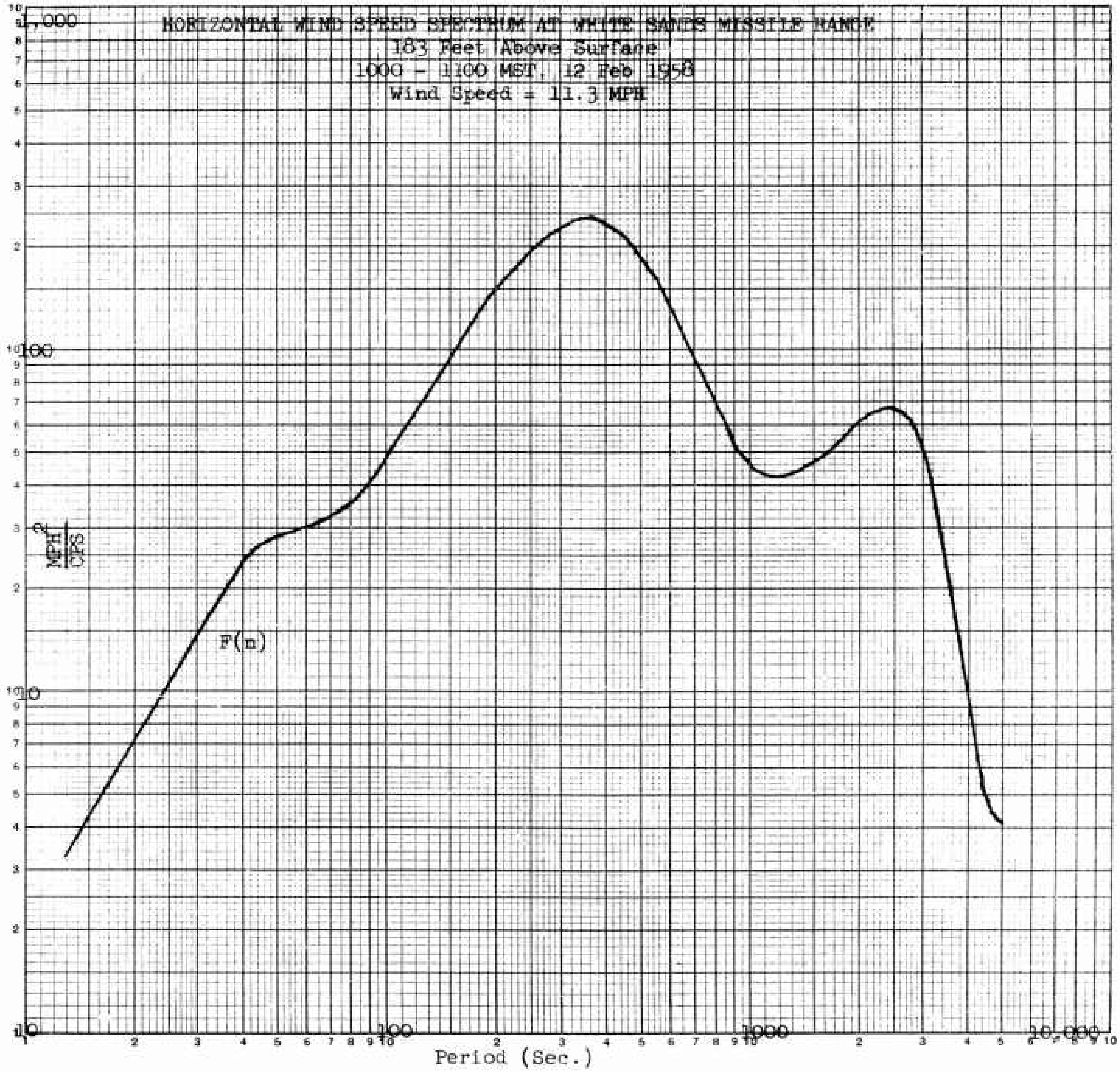

HORIZONTAL WIND SPEED SPECTRUM AT WHITE SANDS MISSILE RANGE
183 Feet Above Surface
1000 - 1100 MST, 12 Feb 1958
Wind Speed = 11.3 MPH
1,000
100
10
1
$\frac{MPH^2}{CPS}$
F(n)
10
100
1000
10,000
Period (Sec.)

Figure 2

Power spectral estimates $F(n)$ are also obtained by use of the auto-covariance function (R_K):

$$R_K = \frac{1}{N - K} \sum_{I=1}^{M-K} X_I X_{I+K},$$

and the Fourier transformation (Ref. 7, 8, 9):

$$F(n_o) = \frac{1}{M}\left[\frac{1}{2}(R_o + R_M) + \sum_{K=1}^{M-1} R_K\right]$$

$$F(n_K) = \frac{2}{M}\left\{\frac{1}{2}\left[R_o + (-1)^n R_M\right] + \sum_{K=1}^{M-1} R_K \cos\frac{\pi K n}{M}\right\}$$

$$F(n_M) = \frac{1}{M}\left\{\frac{1}{2}\left[R_o + (-1)^M R_M\right] + \sum_{K=1}^{M-1} (-1)^K R_K\right\}$$

where

K is the lag number,

M is the maximum lag number used,

N is the sample size,

X IS THE INDIVIDUAL DEVIATION FROM A MEAN,

n IS FREQUENCY DETERMINED BY CHOICE OF K.

THE BANDWIDTH IS GIVEN BY THE EQUATION

$$\Delta n = \frac{(K \pm 1/2)}{2 \Delta T M}$$

AND THE CENTER FREQUENCY IS GIVEN BY

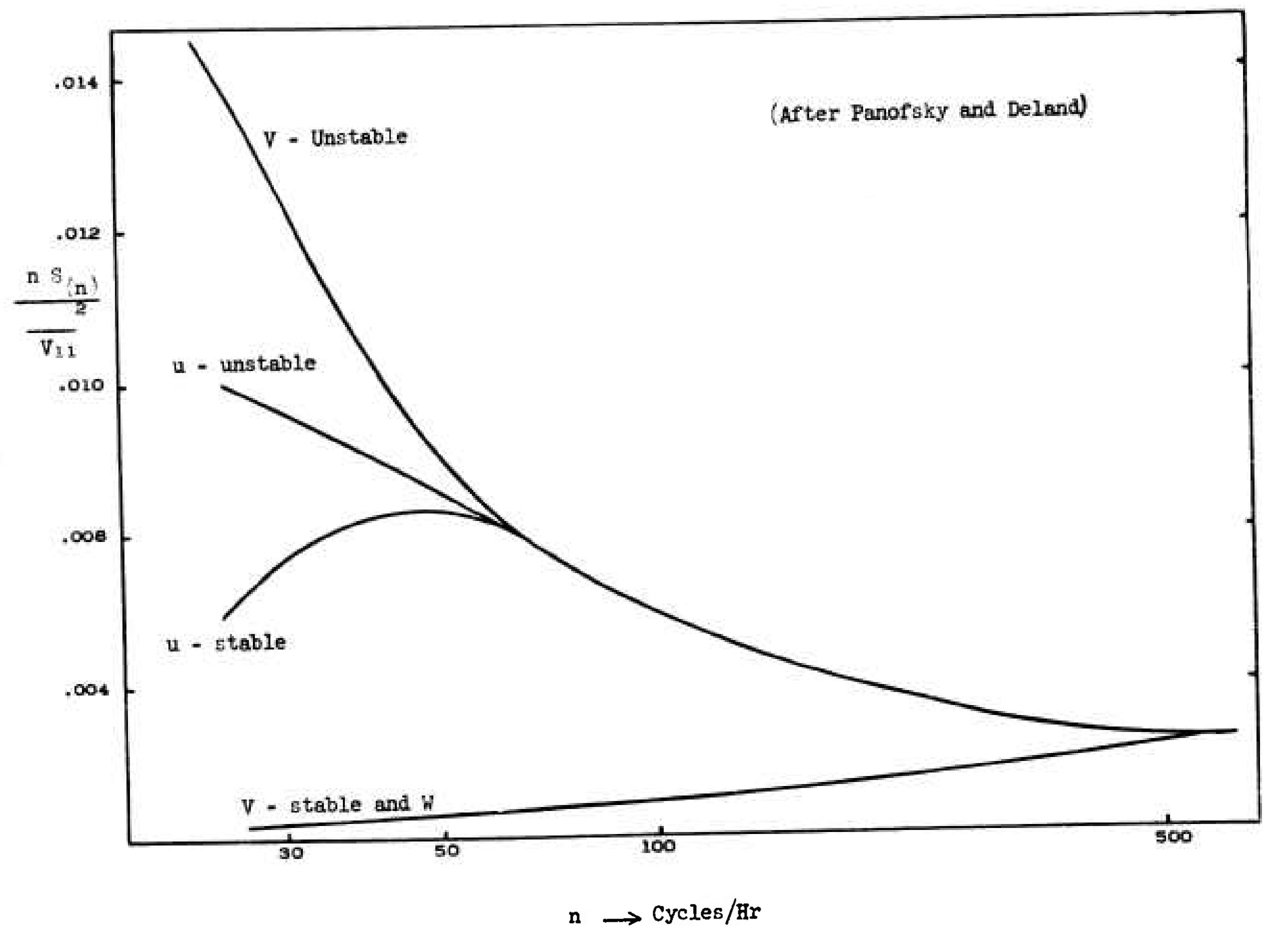
$$n_c = \frac{K}{2M \Delta T}$$

WHERE ΔT IS THE OBSERVATIONAL INTERVAL. THE SPECTRAL ESTIMATES ARE THEN SMOOTHED BY A WEIGHTED MOVING AVERAGE.

PANOFSKY AND DELAND (REF. 10) FOUND FROM SPECTRAL ANALYSIS OF WIND DATA AT BROOKHAVEN, LONG ISLAND, AND O'NEILL, NEBRASKA, THAT, "THE SPECTRUM OF LATERAL VELOCITY COMPONENTS CAN BE DIVIDED MOST CLEARLY INTO LOW-FREQUENCY CONVECTIVE AND HIGH-FREQUENCY MECHANICAL PORTIONS. THE CONVECTIVE PORTION IS ALMOST ENTIRELY A FUNCTION OF LAPSE RATE OR SHORT-WAVE RADIATION, WITH A TENDENCY TO INCREASE WITH HEIGHT. IT IS ESSENTIALLY INDEPENDENT OF WIND SPEED AND GROUND ROUGHNESS. THE MECHANICAL PORTION, ON THE OTHER HAND, IS SENSITIVE TO GROUND ROUGHNESS AND INDEPENDENT OF STABILITY, AND TENDS TO DECREASE WITH HEIGHT. SINCE THE CONVECTIVE PART OF THE SPECTRUM CAN BE LARGE AT DAYTIME, THE TOTAL VARIANCE OF LATERAL VELOCITY SHOWS A TREMENDOUS DIURNAL VARIATION.

"THE PROPERTIES OF THE SPECTRUM OF THE LONGITUDINAL WIND COMPONENT ARE SIMILAR TO THOSE OF THE LATERAL COMPONENT. HOWEVER, THE LOW-FREQUENCY PORTION OF THE SPECTRUM IS CONSIDERABLE EVEN IN STABLE AIR, SHOWING THAT THE LARGEST EDDIES AT NIGHT ARE ELONGATED ALONG THE WIND." FIGURE 3 IS A SCHEMATIC PRESENTATION OF SPECTRA AT O'NEILL. CRAMER (REF. 11) HAS FOUND THAT THE MECHANICAL TURBULENCE, I.E., FREQUENCIES HIGHER THAN 1 CY/MIN, DEPENDS PRIMARILY ON THE SQUARE OF THE MEAN WIND SPEED. CONVECTIVE TURBULENCE, I.E., FREQUENCIES LOWER THAN 1 CY/MIN, DEPENDS UPON THE MEAN WIND SPEED AND THERMAL STRATIFICATION. HE ALSO FOUND, AS THEY DID AT BROOKHAVEN, (REF. 12) THAT THE MAGNITUDE OR STANDARD DEVIATION OF THE FLUCTUATIONS OF THE WIND DIRECTION IS A GOOD INDICATOR OF THERMAL STRATIFICATION AND, THEREFORE, IS A GOOD PREDICTOR OF CONVECTIVE TURBULENCE SPECTRA.

IN THE ATMOSPHERIC TURBULENCE THERE IS NO LOWER FREQUENCY LIMIT SO PRACTICAL LOW FREQUENCY LIMITS MUST BE SELECTED. A LOGICAL CUTOFF VALUE WOULD BE BETWEEN THE CONVECTIVE TURBULENCE AND THE DIURNAL VARIATIONS WHERE THERE IS A MARKED DECREASE IN THE SPECTRAL ENERGY. THIS DECREASE, OR GAP,



Schematic Presentation of Spectra at O'Neill (about 12 m)
Figure 3

OCCURS IN THE FREQUENCY RANGE OF 1 TO 2 CYCLES PER HOUR AS INDICATED IN FIGURE 4 (REF. 13). FREQUENCIES LOWER THAN THOSE PRODUCED FROM CONVECTIVE ENERGY WILL NOT BE CONSIDERED IN THIS PAPER.

CORRELATION OF WIND BETWEEN TWO POINTS

CROSS-SPECTRAL ANALYSIS DETERMINES THE CORRELATION OF VARIOUS FREQUENCIES BETWEEN TWO TIME SERIES AND IS COMPOSED OF TWO PARTS, THE COSPECTRUM AND THE QUADRATURE SPECTRUM. THE COSPECTRUM PORTION CONSIDERS THE SIMULTANEOUS RELATIONS BETWEEN TWO SERIES AND THE QUADRATURE SPECTRUM CONSIDERS THE RELATIONSHIP BETWEEN TWO SERIES WHEN ONE IS OUT OF PHASE BY A QUARTER PERIOD.

THE COHERENCE FUNCTION, WHICH IS ANALOGOUS TO THE SQUARE OF THE CORRELATION COEFFICIENT, EVALUATES THE RELATIONSHIP BETWEEN TWO TIME SERIES FOR VARIOUS FREQUENCIES AND IS DEFINED BY

$$COH_{(n)_{X/Y}} = \frac{Q^2_{(n)} + C^2_{(n)}}{F_{(n)_X} F_{(n)_Y}}$$

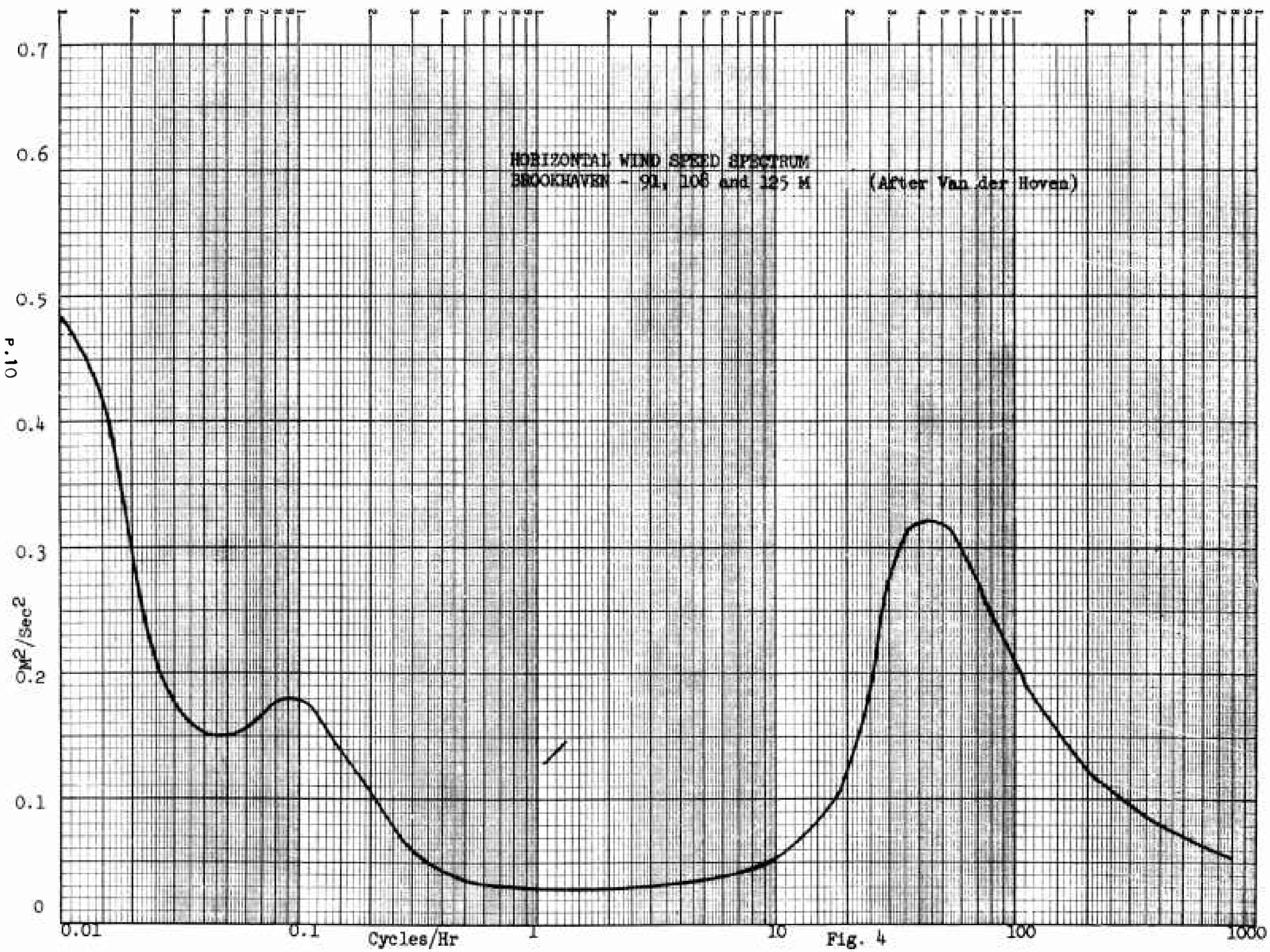
WHERE

$C_{(n)}$ IS THE COSPECTRUM BETWEEN THE TWO TIME SERIES X AND Y AND

$Q_{(n)}$ IS THE QUADRATURE SPECTRUM BETWEEN THE SAME TWO SERIES. FOR COMPUTATIONAL PROCEDURES OF THE CROSS SPECTRUM SEE REFERENCES 5, 8, 9, OR 10.

BROOKHAVEN HAS APPLIED THE CROSS-SPECTRAL EQUATIONS TO WIND PROFILE PREDICTION TECHNIQUES (REF. 14-17) USING THE 75-, 150- AND 300-FOOT LEVELS ON ONE TOWER (ACE) AND THE 150-FOOT LEVEL ON A SECOND TOWER (KING) LOCATED 900 FEET AWAY FROM THE FIRST. TABLES I AND II ARE COHERENCE VALUES COMPUTED AT BROOKHAVEN (REF. 15) WHERE THE WIND WAS BLOWING DIRECTLY FROM ONE TOWER TO THE OTHER. BOTH RUNS WERE UNDER LAPSE CONDITIONS WITH THE AZIMUTH FLUCTUATION RANGING FROM 15° TO 45°.

(CONT. ON PAGE 13)



HORIZONTAL WIND SPEED SPECTRUM
BROOKHAVEN - 91, 108 and 125 M
(After Van der Hoven)
0.7
0.6
0.5
0.4
0.3
0.2
0.1
0
M^2/Sec^2
0.01
0.1
1
10
100
1000
Cycles/Hr

Fig. 4

P.10

TABLE I

COHERENCE VALUES - HIGHER WIND SPEEDS

$\bar{U}$ 150* FT = 6.55 M/SEC, $\bar{U}$ 150 FT = 6.02 M/SEC, $\bar{U}$ 300 FT = 6.58 M/SEC

FREQ (CY/HR)	COHERENCE 150* - 150	(AFTER SINGER) 150* - 300	150 - 300 (FT)
0	0.25937	0.15361	0.18454
5	0.77970	0.64064	0.75488
10	0.46451	0.57854	0.56491
15	0.38664	0.32281	0.38890
20	0.19913	0.18186	0.24469
25	0.19944	0.20020	0.32738
30	0.51674	0.21136	0.33861
35	0.41642	0.32838	0.38582
40	0.25262	0.40498	0.27840
45	0.09653	0.19290	0.20341
50	0.06619	0.08499	0.20543
55	0.07004	0.07251	0.12810
60	0.15204	0.07101	0.18397
80	0.13694	0.03565	0.04197
100	0.04159	0.05613	0.00115
120	0.16702	0.05615	0.03449
140	0.07087	0.04435	0.07055
160	0.04970	0.02469	0.00059
180	0.02866	0.07614	0.08271

* KING TOWER.

TABLE 11

COHERENCE VALUES - LOWER WIND SPEEDS

$\overline{U}$ 150* FT = 3.71 M/SEC, $\overline{U}$ 150 FT = 3.67 M/SEC, $\overline{U}$ 300 FT = 3.70 M/SEC

FREQ (CY/HR)	COHERENCE 150* - 150	(AFTER SINGER) 150* - 300	150 - 300 (FT)
0	0.32445	0.18109	0.27907
5	0.73456	0.61661	0.78931
10	0.57689	0.44867	0.51747
15	0.20045	0.04538	0.26880
20	0.22248	0.06281	0.27279
25	0.16161	0.07387	0.04009
30	0.06685	0.04097	0.00957
35	0.00713	0.15390	0.00204
40	0.04656	0.12309	0.00404
45	0.17881	0.11434	0.00005
50	0.16562	0.07509	0.04376
55	0.03171	0.07824	0.02927
60	0.05403	0.08705	0.01995
80	0.03207	0.11220	0.12661
100	0.05224	0.14466	0.09286
120	0.11845	0.09259	0.17874
140	0.09658	0.06787	0.12745
160	0.01974	0.02632	0.11042
180	0.00346	0.	0.00905

* KING TOWER

Table III, also extracted from Reference 15, consists of coherence values between three levels on a single tower under a nighttime (temperature inversion) condition. The direction trace for this run approximates a smooth curve.

Cramer (Ref. 5) analyzed the data obtained from Project Prairie Grass for correlations of the u (downwind) and v (crosswind) fluctuations of the wind at a height of two meters. He determined that the quadrature spectral estimates were not statistically significant and so computed the correlations (R_C) by the equation

$$R_{C(n)} = \frac{C_{(n)}^{2}}{F_{(n)_1} F_{(n)_2}}$$

Figure 5 (Cramer, Ref. 5) presents correlation coefficients vs separation distance for the various periods given in Table IV. This figure represents samples where the wind was blowing parallel to the sensor array.

Coherence values between various levels on the White Sands meteorological tower during daytime (lapse) conditions are presented in Table V. The power spectra for this run are given in Figures 1 and 2.

Some other approaches to relate wind measurements at one point to those at another point in space are discussed by: Howcraft and Smith (Ref. 18); Cramer, Record, and Vaughn (Ref. 19); Lamberth and Veith (Ref. 20); Durst (Ref. 21); and Court (Ref. 22).

APPLICATION OF WIND SPECTRA INFORMATION TO BALLISTICS

Zbrozek (Ref. 23) and Thorson and Bohne (Ref. 24) discuss the output power spectrum of an aircraft or missile $[F_M(\Omega)]$ as a function of the power spectrum of turbulence $[F(\Omega)]$ and the aircraft frequency response function $[T(\Omega)]$.

The output spectrum can be calculated by the relationship

$$F_M(\Omega) = [T(\Omega)]^2 \times F(\Omega)$$

(Cont on Page 17)

TABLE III

COHERENCE VALUES - TEMPERATURE INVERSION CONDITIONS

$\bar{U}$ 75 FT = 3.44 M/SEC, $\bar{U}$ 150 FT = 6.33 M/SEC, $\bar{U}$ 300 FT = 9.01 M/SEC

FREQ (CY/HR)	COHERENCE 75 - 150	(AFTER SINGER) 75 - 300	150 - 300 (FT)
0	0.44598	0.13006	0.14846
5	0.60920	0.17281	0.31483
10	0.37815	0.35140	0.35924
15	0.17952	0.09043	0.15680
20	0.28390	0.06399	0.14721
25	0.13622	0.02686	0.07647
30	0.07008	0.00058	0.12540
35	0.14003	0.00111	0.12700
40	0.13978	0.00489	0.11364
45	0.09225	0.02359	0.05435
50	0.24533	0.12522	0.00106
55	0.08598	0.04293	0.15396
60	0.03975	0.04514	0.01404
80	0.00212	0.02632	0.07168
100	0.10833	0.11111	0.09259
120	0.06974	0.06579	0.05625
140	0.03137	0.02941	0.10667
160	0.02564	0.06410	0.02778
180	0.01070	0.02941	0.09191

TABLE IV

CENTRAL FREQUENCIES FOR SELECTED LAG VALUES

K	$T_c = \frac{1}{n_c}$ (SEC)
1	128
2	64
3	43
4	32
5	26
6	21
8	16
10	12.8
12	10.7
15	8.5
20	6.4
24	5.3

Central frequencies of frequency intervals associated with selected values of K (lag) used in obtaining scale estimates; for convienience, data are inverted and expressed in terms of period rather than frequency.

TABLE V

COHERENCE VALUES DURING LAPSE CONDITIONS - WSMR

COHERENCE (U COMPONENT)

K	PERIOD (SEC)	15 - 63	15 - 111	15 - 203	63 - 111	63 - 203 (FT)
1	240	.7006	.7334	.2549	.9173	.7066
2	120	.5871	.3781	.1146	.7517	.5531
3	80	.7257	.3762	.3522	.6187	.3540
4	60	.7463	.2596	.5112	.4579	.5177
5	48	.3307	.0066	.1585	.1559	.3720
6	40	.2991	.0355	.0300	.0914	.0553
8	30	.0533	.5162	.4415	.3262	.0066
10	24	.0308	.5389	.0423	.0029	.3072
12	20	.2638	.2560	.1863	.0962	.2250
16	15	.0053	.2938	.1622	.0260	.1437
			COHERENCE (V COMPONENT)			
1	240	.2200	.1547	.2064	.9809	.9945
2	120	.0945	.0999	.0271	.8032	.4794
3	80	.0281	.0468	.1488	.3229	.0318
4	60	.2019	.1410	.1107	.5752	.0853
5	48	.0801	.0230	.4179	.3273	.0128
6	40	.0586	.0900	.6101	.2747	.0097
8	30	.0001	.2164	.3395	.1626	.1340
10	24	.1214	.1675	.2121	.3859	.1157
12	20	.1556	.1995	.0229	.0335	.2778
16	15	.0438	.0562	.1303	.0655	.4364

where

Ω = space frequency or wave number. For wind field characteristics along a horizontal path $[F(\Omega), \Omega]$ curves may be estimated using Taylor's hypothesis and available $[F_{(n)}, n]$ curves.

Approximate $[F(\Omega), \Omega]$ curves for a vertical path may be directly estimated from pilot balloon wind measurements.

When the admittance function, i.e., response to a particular space frequency, of a rocket is known we can determine the bandwidth of the turbulence spectrum that will cause the missile to deviate from its expected trajectory. For example, suppose an overdamped missile with a natural wavelength of 1,000 feet is fired horizontally and has an admittance function described by Figure 6. Using the turbulence spectral values in Figure 1 with the assumption that Taylor's hypothesis holds true, and the admittance ratio values in Figure 6 we obtain an effective spectral curve $[T(\Omega), \Omega]$ as seen in Figure 7. From this curve we can determine the highest frequencies that affect the missile dispersion pattern. Smoothing and filtering techniques may be employed to eliminate the frequencies where the energy level is so low that it is insignificant. Smoothing and filtering of data is discussed by Holloway (Ref. 25).

Space spectral information, when introduced into ballistic equations, will describe the effect of low level wind on the dispersion pattern of missile trajectories. Ballistic equations, using the turbulence spectra, are being developed by Rachele (Ref. 26) and Walter (Ref. 27).

When a missile is to be laun[illegible], low level wind is measured at a point or points in space some d[illegible]e from the launcher and the anticipated missile path. This wind information is used to correct the launcher azimuth and elevation for the wind effects and is also used for post firing analysis of the flight.

A method of predicting the instantaneous wind at the launcher from some remote point is to filter out all frequencies which have no significant correlation between the two points while retaining all of the frequencies with significant correlation. The coherence function, previously discussed in this report, is a measure of the correlation coefficient for any particular frequency. When the coherence values are known for the separation distance between the sensor and the launcher the uncorrelated high frequencies can then be filtered out of the data by use of a moving average of period T_c.

A moving average will filter out the turbulence due to frequencies higher than $1/(2T_c)$. For example, from Figure 5 and Table IV we see that, under ideal daytime conditions, an averaging time of about 26 seconds is necessary to obtain correlations of greater than 0.7 for a horizontal

(Cont. on Page 21)

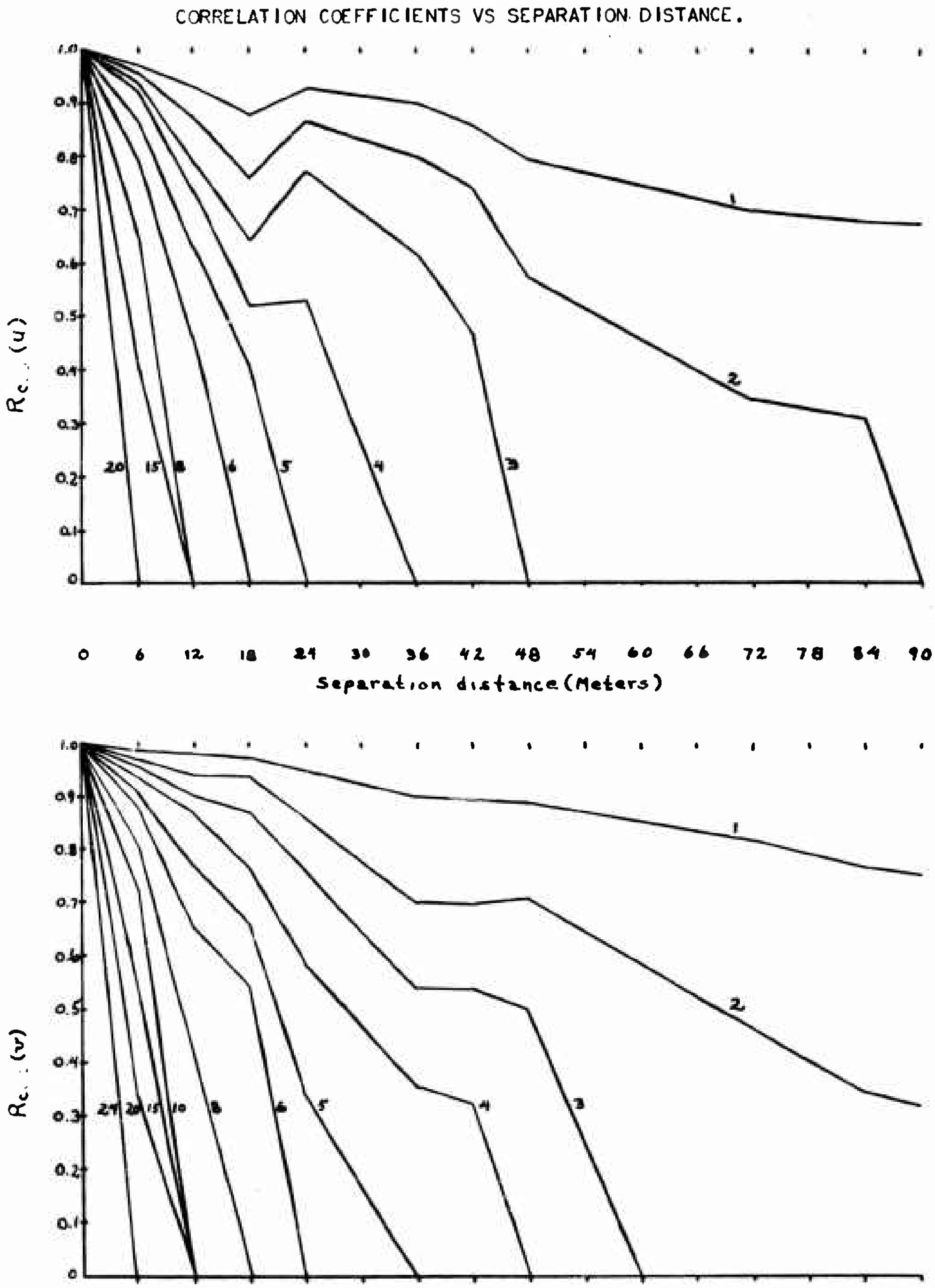
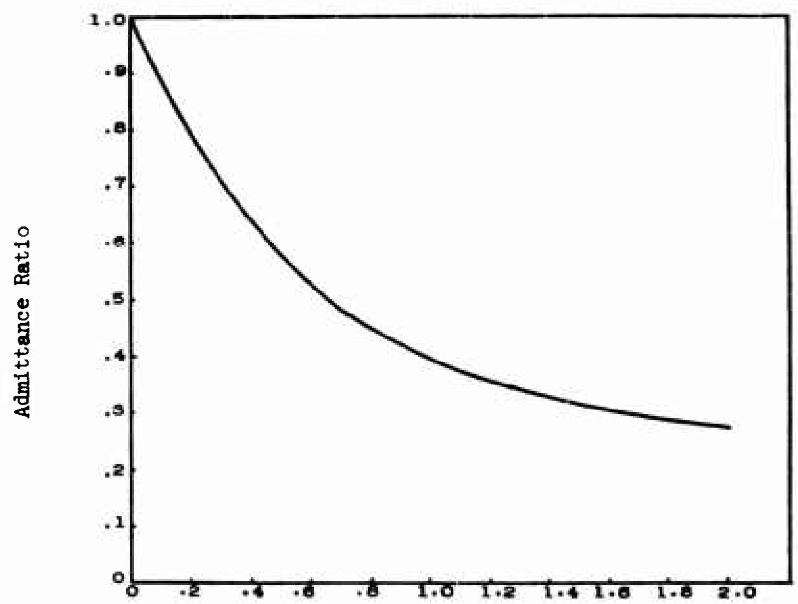

CORRELATION COEFFICIENTS VS SEPARATION DISTANCE.
Rc.. (u)
Rc.. (v)
Separation distance (Meters)
0 6 12 18 24 30 36 42 48 54 60 66 72 78 84 90
1.0 0.9 0.8 0.7 0.6 0.5 0.4 0.3 0.2 0.1 0
1 2 3 4 5 6 8 10 15 20 24

Figure 5



Rocket Wavelength
Eddy Wavelength

ADMITTANCE FUNCTION.

Figure 6

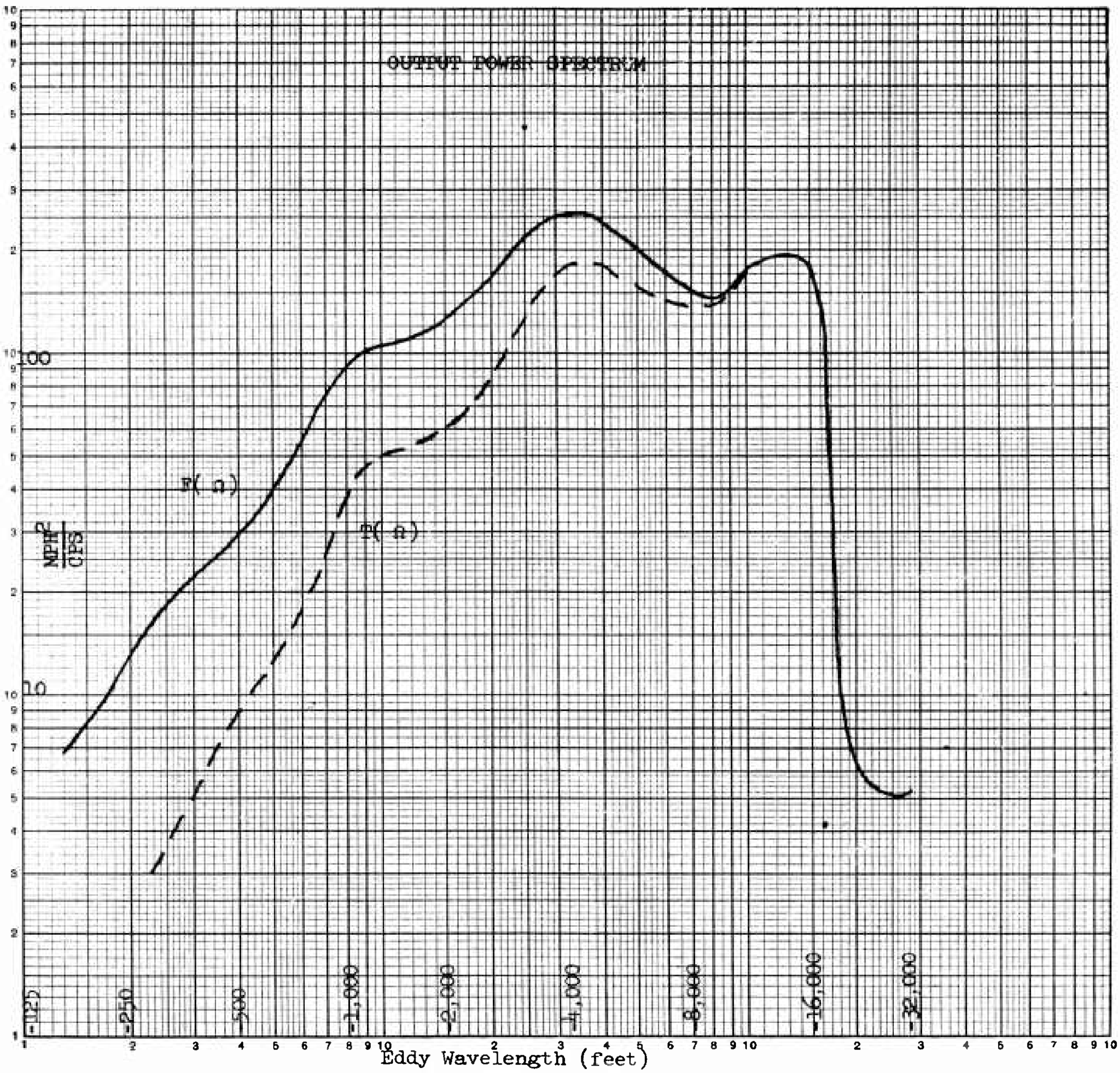

OUTPUT POWER SPECTRUM
F(n)
T(n)
100
10
MPH²/CPS
125
250
500
1,000
2,000
4,000
8,000
16,000
32,000
Eddy Wavelength (feet)

Fig. 7

SEPARATION DISTANCE OF 50 FEET. TABLES I AND II INDICATE THAT AN AVERAGING TIME OF 5 TO 10 MINUTES IS NECESSARY TO GET A RELIABLE ESTIMATE OF THE WIND FOR A HORIZONTAL SEPARATION DISTANCE OF 1,000 FEET. THE COHERENCE FUNCTION IS ALSO APPLICABLE TO ESTIMATE THE INSTANTANEOUS WIND IN A VERTICAL PATH. TABLES I AND II INDICATE THAT AN AVERAGING TIME OF APPROXIMATELY 5 MINUTES AT 150 FEET IS NECESSARY TO PREDICT THE WIND SPEED AT 300 FEET.

CONCLUSIONS

IT IS FELT THAT LOW LEVEL WIND PREDICTION SCHEMES FOR LAUNCH TIME ESTIMATES OF IMPACT DISPERSION PATTERNS CAN BE IMPROVED OVER CURRENT METHODS BY USE OF THE SPECTRAL AND CROSS-SPECTRAL ANALYSIS OF THE ATMOSPHERE. SIMPLE AND PRACTICAL METHODS OF PREDICTING THE TURBULENCE FIELD FOR FIELD USE HAVE BEEN DEVELOPED FOR PARTICULAR LOCATIONS AND CAN BE ADAPTED FOR ANY LAUNCH SITE IN QUESTION.

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